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Precision analysis of ^{198}Au beta spectral shape

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Abstract. The investigation of the 965 keV transition of ^{198}Au shows that the same data analysed with the Fermi function taken from NBS tables and that from Dzhelepov and Zyrianova (corrected for finite size) differ by 3% in slope. With a shape factor $C(W) = k(1+aW)$ the coefficient a has been determined to be $-0{\cdot}022 \pm 0{\cdot}011$ using Bhalla and Rose tables with Buhring's screening correction and the correct end point determined from the proper physical behaviour of the shape factor in the neighbourhood of the end point energy. The use of this coefficient through the PBS method gives $\alpha_{412\mathrm{K}} = (301 \pm 3)10^{-4}$ in agreement with theory as well as the recent very accurate IEC methods. This means that the disagreement of the PBS method with theory and other accurate methods arises mainly through the occurrence of a shape factor in the electron energy spectrum and the use of the correct shape factor in the PBS method gives good agreement.

1. Introduction

The decay scheme of ^{198}Au is well established (*Nuclear Data Sheets* 1966). The main beta transition (99%) of 0·966 MeV maximum energy which leads from the ^{198}Au ground state (2^-) to the 0·412 MeV excited state (2^+) of ^{198}Hg is a first-forbidden nonunique transition with a lg *ft* value of 7·3. The condition for the applicability of the ξ approximation that $\xi \gg W_0 (\xi = 16{\cdot}2,\ W_0 = 1{\cdot}9)$ being well fulfilled in this case, beta decay observables associated with this transition should exhibit energy independence to within $1/\xi \simeq 6\%$. The beta–gamma directional and circular polarization correlations were performed by Steffen (1960) and Rao *et al* (1967) and they found that these parameters are proportional to p/W and that $V_0 \simeq Y_1 \gg z \simeq x \simeq u \simeq y \simeq w$ consistent with the ξ approximation. But there has been considerable disparity as to the experimental shape of the above transition. Some indicate that the deviation from the statistical distribution is as large as 10% while others report shape factors which are nearly statistical. The situation has been similarly unsatisfactory with respect to the K conversion coefficient of the 412 keV transition in ^{198}Hg which is a pure E2 transition. The values obtained by IEC methods were in good agreement with the theoretical value, whereas the comparison of conversion peak and beta spectrum as measured in a magnetic spectrometer (PBS method) gave consistently low values. It is clear that the presence of a shape factor in the expression for the continuum would affect the value obtained for α_K. If the negative slope of the shape factor is overestimated for any reason, then the spectrum so obtained simulates too many low energy electrons and the conversion coefficient by the PBS method would be small.

Most of the earlier discrepancies stem from the wrong choice of the end point energy. As follows from the paper of Paul (1965), the result for the shape factor obtained depends on the Fermi function which is taken. The allowed shape factor L_0 for ^{198}Au calculated from the tables of Bhalla and Rose (1962) varies from 0·811 to 0·789 in the energy range used here. Various distorting effects such as spectrometer scattering, spectrometer and detector backgrounds, detector backscattering etc may be responsible for the remaining discrepancy. Recently Spejewski (1966) has measured the shape of this transition in a 4π semiconductor beta and gamma coincidence arrangement. With the shape factor $C(W) = k(1+aW)$, the range of 'a' obtained in that arrangement, namely, $-0{\cdot}118 < a < -0{\cdot}017$ is as large as the existing discrepancy in the reported results for this transition. The reason for the poor performance of the 4π detector is discussed elsewhere (Nagarajan 1968).

2. Details of analysis

The K conversion coefficient of the 412 keV transition has been measured by other methods (Pettersson *et al* 1961, Lewin 1965, Hubert 1951) and recently very accurately by the IEC method (Bergkvist and Hultberg 1964). The values are in excellent agreement with the theoretical values (Rose 1958, Sliv and Band 1956). Hence a measurement of the conversion coefficient of this transition can provide an independent check on the performance of the spectrometer. A Siegbahn–Slatis spectrometer equipped with well-type plastic is used in the present work. The optimization of baffle system and detector arrangements are described elsewhere (Nagarajan *et al* 1969a, 1969b). The ^{198}Au source was obtained from the Bhabha Atomic Research Centre, Trombay, in the form of chloride in solution with high specific activity. Vacuum evaporated sources of 10–20 μg cm^{-2} thick on 180 μg cm^{-2} Al foils were used. Sources were also prepared by evaporation of a drop of the active $AuCl_3$ solution on aluminized Mylar foils. Insulin was used to define the source area. All sources were 2 mm in diameter and they were properly centred.

The decay scheme (*Nuclear Data Sheets* 1966) of ^{198}Au is shown in figure 1. The relative intensities of the beta groups were deduced from the gamma intensity measurements of Keeler and Connor (1965). The presence of ^{199}Au activity was checked by looking for the 208 K line in ^{199}Hg. From the recent decay scheme (*Nuclear Data Sheets* 1966), the intensity of the 208 K line should be 1/16·5 of the total activity. In all different consignments of the sources obtained, the ^{199}Au activity was less than 2% of ^{198}Au activity.

The spectrum was scanned up to 1350 keV roughly in steps of 24 keV, taking the background at every measurement point. The various corrections including the decay correction were performed by the computer analysis. In addition to the use of the tables of Bhalla and Rose (1962), the NBS tables (Feister 1952) and the tables of Dzhelepov and Zyrianova (1956) were also used to determine the effect of different Fermi functions.

The intensity of the ground state transition to ^{198}Hg is estimated to be 0·03% of the principal beta group and this weak high energy component was subtracted from the gross spectrum. The subtraction of this outer group did not have any effect on the 966 keV continuum.

The energy region above 420 keV was subjected to the shape factor analysis. Thus the interferences from the K and L+M lines of 412 keV transition, the low energy beta group of ^{198}Au (E_0 = 284 keV) as well as the beta groups of ^{199}Au (E_0 of the most energetic group = 458 keV) are avoided. In the first run a homogeneous liquid-deposited

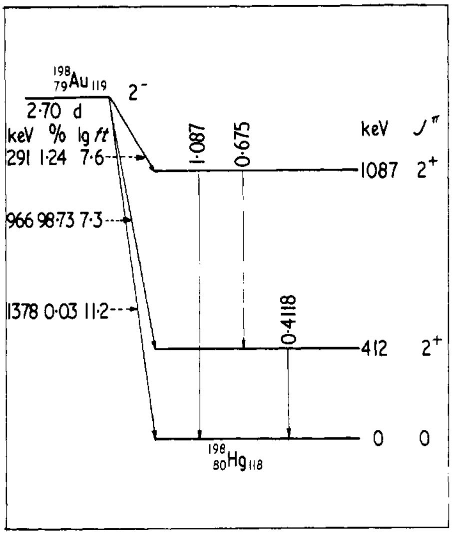

Figure 1. Decay scheme of ^{198}Au.

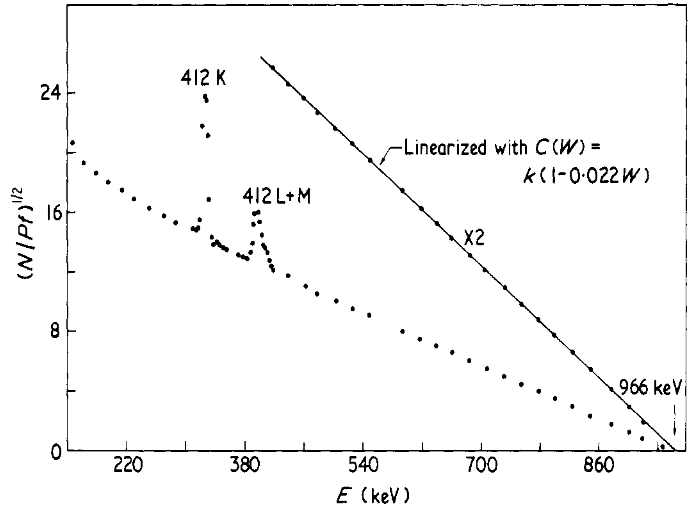

Figure 2. Fermi–Kurie plot of the beta spectrum of ^{198}Au.

source was used and the analysis (figure 3) was done with the Fermi function taken from NBS tables. An exact resolution correction (Nagarajan *et al* 1969a, 1969b) is applied without making any assumptions about the lineshape

$$C_R = 1-\left(0{\cdot}0001692\frac{p}{N}\frac{dN}{dp}+0{\cdot}00002539\frac{p^2}{N}\frac{d^2N}{dp^2}\right).$$

The lineshape parameters in the above expression, as well as the transmission and resolution of the spectrometer, were found to be energy independent, indicating the absence of any scattering in the spectrometer or source.

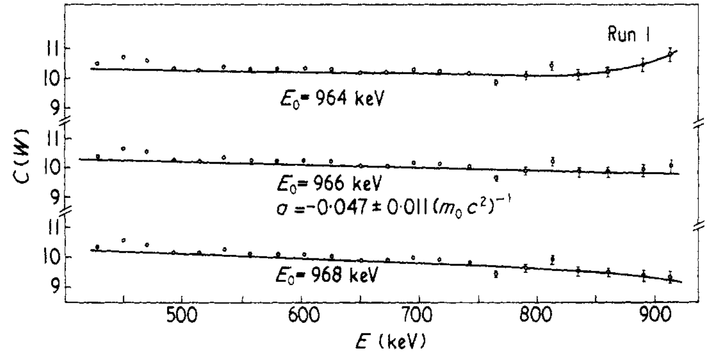

Figure 3. Shape factor plot of the 966 keV beta transition of ^{198}Au. The choice of the correct W_0 from the proper physical behaviour of $C(W)$ near W_0 is shown.

The dependence of the decay electron energy distribution on β moments occurs only in a term called the 'shape factor $C(W)$' and it is the careful measurement and analysis of this 'shape factor' which may yield important information about β moments as well as the nuclear spin and parity changes that have taken place during the transition. The β moments play only a perturbation role in the determination of the electron energy distribution and the subtleties of the beta shape may often be lost sight of, since the experimental shape is extremely sensitive to distortions in the data. The experimental shape of a beta group is given by

$$C(W) = \frac{N}{pf(W_0 - W)^2}$$

where N is the counting rate at momentum p. The reduced Fermi function $f(Z, p)$ is defined by

$$f(Z, p) = p^2 F = p^2 F_0 L_0 = \tfrac{1}{2}(f_1^2 + g_{-1}^2)(\text{sc}).$$

The different methods used in shape factor analysis and their relative merits are discussed by Nagarajan (1968) and Nagarajan *et al* (1970). The present procedure contains two steps—the first for determining the end point W_0 which is physically correct and the second for finding the coefficients of the shape factor $C(W)$. In determining W_0, one notices that the shape factor varies as $N(W)/(W_0 - W)^2$ in the small region near W_0. Since $N(W)$ as well as $(W_0 - W)^2$ goes to zero as W approaches W_0, the shape factor is extremely sensitive to the choice of W_0. Hence the shape factor goes rapidly to infinity or to zero and this behaviour in the neighbourhood of the end point can be used to determine the true end point. This method is more reliable and physically more significant. A program called 'BETASHAP' finds the approximate end point by extrapolating the Fermi–Kurie plot. It starts at an end point less than this approximate value by 20 keV and steps up W_0 by 2 keV each time. For each value of W_0, it computes $N/pf(W_0 - W)^2$ against W and plots all 20 shape factor curves simultaneously on the same figure. From a visual examination of these curves, one can easily choose the one with the correct end point. In figure 3, these curves are presented, one for the correct end point W_0 and the neighbouring two curves on either side. The second step is to compute $C(W) = N/pf(W_0 - W)^2$ for the correct end point energy and the statistical error in

$C(W)$. For details see Nagarajan (1968) and Nagarajan *et al* (1970). A program 'SHAPE-FIT' evaluates k and a by a weighted least squares fitting of $C(W) = k(1+aW)$. A standard procedure is adopted (Nagarajan 1968, Nagarajan *et al* 1970) for this purpose.

The correct end point is thus found to be 966 ± 2 keV. Two further measurements were carried out with thin vacuum sublimited sources and they were analysed respecrespectively with NBS tables (Feister 1952) and the tables of Dzhelepov and Zyrianova (1956) (figure 4). The first run was reanalysed with the exact functions of Buhring (1963). The screening factor on F_0L_0 at 640 keV is 1·8 %. With an assumed shape factor $C(W) = k(1+aW)$, the reanalysis of run 1 data gave $a = -0{\cdot}022 \pm 0{\cdot}011\ (m_0c^2)^{-1}$. The coefficients a and W_0 obtained from an independent analysis of each run are given in table 1.

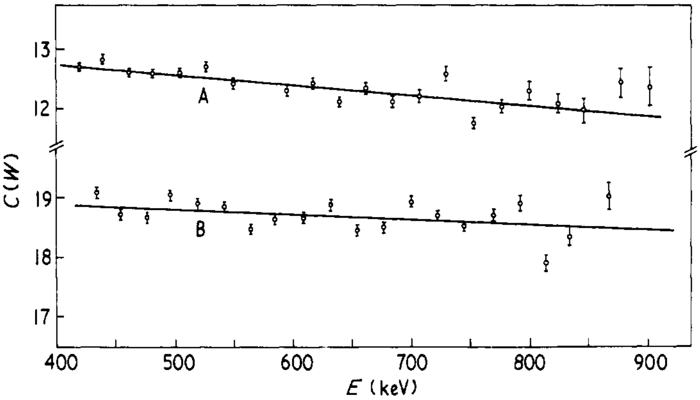

Figure 4. Shape factor of the ^{198}Au beta transition for different runs. A, run 2, analysed with the Fermi functions of NBS tables, $E_0 = 966$ keV, $a = -0{\cdot}064+0{\cdot}01\ (m_0c^2)^{-1}$. B, run 3, analysed using $f = F_0L_0$ of Dzhelepov and Zyrianova, $E_0 = 966$ keV, $a = -0{\cdot}025 \pm 0{\cdot}008\ (m_0c^2)^{-1}$.

Table 1. Result on the shape factor of the 966 keV beta transition of ^{198}Au

Run number and nature of the source	Tables used	E_0 (keV)	a $(m_0c^2)^{-1}$
1 (liquid deposited)	NBS	966 ± 1	−0·047 ± 0·011
	Bhalla and Rose with Buhring's screening correction	966 ± 1	−0·022 ± 0·011
2 (vacuum evaporated) 10 μg cm^{-1}	NBS	966 ± 1	−0·064 ± 0·01
3 (vacuum evaporated) 15 μg cm^{-1}	Dzhelepov and Zyrianova	966 ± 1	−0·025 ± 0·008

All the three runs consistently give 966 ± 1 keV for the end point of the β group feeding the 412 keV level of ^{198}Hg. This leads to an accurate Q_β value of 1378 ± 1 keV for the decay of ^{198}Au. These results are incorporated in the decay scheme (figure 1).

Table 2 summarizes the end points obtained by various experimental workers for this transition. It is surprising that our result differs from many of the recent authors. Most of them estimate the end point from the uncorrected Fermie–Kurie (FK) plot which should yield lower values of W_0 for a spectrum with a negative shape factor coefficient. Even in cases where the shape factor is computed using an end point determined from an uncorrected FK plot and the FK plot is linearized with this shape factor, the linearized FK plot will extrapolate itself to values of W_0 close to its original choice only. Besides this, many of the authors neglect the fact that, in a precise determination of the end point, it is essential to apply an exact resolution correction and correct for the background. The LKB Slatis–Siegbhan β ray spectrometer has the advantage that its annular slit can be closed from outside and reproduced at will. This enabled us to determine the source dependent background at every measurement point. In most reported literature, the background spectrum is constructed from the count rate at zero field and the

Table 2. Shape factor of the main beta transition of ^{198}Au and α_{412K} measured by the PBS method by different authors

Authors	Instruments	E_0 (keV)	a $(m_0c^2)^{-1}$	$\alpha_K \times 10^4$	Fermi function
Wapstra *et al* (1958	Short lens	966 ± 3	−0·11 ± 0·017†	281 ± 5	NBS
de Vries *et al* (1960)	Double focus	968 ± 3	−0·134 ± 0·016†	241 ± 9	NBS
Depommier and Chabre (1961a and b)	Interm. image	962 ± 1	−0·062 ± 0·007†		NBS
Graham (1961)		964 ± 4	−0·046 ± 0·01†	289 ± 19	?
Hamilton *et al* (1962)	180° inhomog.	960 ± 5	$1-0{\cdot}33W \pm 0{\cdot}074W^2$	282 ± 10	NBS
Sharma *et al* (1962)	Interm. image	957 ± 5	−0·02		?
Newbolt and Hamilton (1965)	Double lens		$1-0{\cdot}03W+0{\cdot}074W^2$	286 ± 10	?
Lehmann (1964)	Interm. image	965 ± 2	−0·155 ± 0·015		?
Parsinault (1965)	Uniform field	960 ± 2	$1-0{\cdot}33W+0{\cdot}068W^2$	283 ± 10	?
Lewin (1965)	Long lens	962 ± 2	−0·014 ± 0·024		NBS
Spejewski (1965	(Semi 4π)	?	$-0{\cdot}118 < a <$ $-0{\cdot}017$		?
Keller and Connor (1965	Interm. image	960 ± 1	0·002 ± 0·006 (L_0 included) −0·007 ± 0·006 (L_0 not included)	299 ± 4	NBS
Paul (1965)	Interm. image	961 ± 1·2	−0·057 ± 0·006 (L_0 not included) −0·031 ± 0·006 (L_0 included)	299 ± 2	NBS Dzhelepov and Zyrianova
Beekhuis and De Waard (1965)	Interm. image	962 ± 2	−0·05 ± 0·011		NBS
Present work	Interm. image	966 ± 1	−0·047 ± 0·011 (L_0 not included) −0·022 ± 0·011 (L_0 included)	301 ± 3	NBS Bhalla and Rose and Buhring

† Recalculated from the g value of Wapstra's expression $C(W) = k\{1+g(W_0-W)\}^2$ using $a = -2g/(1+2gW_0)$.

spectrometer current-setting beyond the end point. This procedure is ambiguous, because the source dependent background arising from photons varies with the current-setting.

It is also worth noting that Keeler and Connor (1965) and Beekhuis and De Waard (1965) recognize the sensitive dependence of the shape factor coefficients a, b and c on W_0. Keeler and Connor (1965) merely report the values of a, b and c for various values of W_0 and choose a, b, c for the end point determined from the uncorrected FK plot. Also the necessity of fitting the shape with the W^2 term, as done by Keeler and Connor (1965), will not arise if the shape factor itself is made to behave well near W_0 by the proper choice of W_0 (Nagarajan *et al* 1971).

Beekhuis and De Waard (1965), on the other hand, treat W_0, a, b and c as free parameters and the experimental quantity $y = N/pf$ is least square fitted with

$$y = k(1+aW+b/W+cW^2)(W_0-W)^2$$

demanding that χ^2 is minimum. In this method, the least squares minimization may lead to a W_0 different from its physical value and, in compensation, $C(W)$ is allowed to go to small or large values.

Our results on the shape coefficients 'a' with and without the allowed shape factor L_0 whose energy dependence is fixed for a given transition, agree with the recent authors (viz Paul 1965, Beekhuis and De Waard 1965). The statistical shape reported by Keeler and Connor (1965) may be in error since they use a flat plastic detector which has large backscattering and the baffles may not have been adjusted for minimal scattering. We have used a plastic well of cylindrical geometry whose backscattering effect is 0·2% at 80 keV and the counting efficiency is unity down to 50 keV (Nagarajan *et al* 1969a, 1969b). The baffles, the source and the detector are thoroughly checked for distortionless investigation (Nagarajan *et al* 1969a, 1969b).

3. Determination of α_K for the 966 keV beta transition

The FK plot of the beta continuum was linearized with the shape factor

$$C(W) = k(1-0{\cdot}022W)$$

above 450 keV. This linearized FK plot (figure 2) was extrapolated to low energies and the true spectrum was reconstructed from this linear FK plot. The K/(L+M+N...) ratio was also estimated to be 2·36±0·6.

If the weak ground-state beta transition is neglected and if x, $1-x$ are the relative intensities of the 291 keV and 966 keV beta groups respectively, then the intensity of the 412 keV γ rays is

$$I_{412} = \frac{A_{966}}{1-x} - I_{1087} - (A_{\mathrm{K}} + A_{\mathrm{L+M+N}}) \tag{1}$$

where A_{K} and A_{966} are the areas of the K line of the 412 keV transition and of the 966 keV beta continuum respectively. $A_{\mathrm{L+M+N}}$ is the area of unresolved L+M+N... lines. Relation (1) yields

$$I_{412} = \frac{A_{966}/(1-x) - (A_{\mathrm{K}} + A_{\mathrm{L+M+N}})}{1 + I_{1087}/I_{412}}$$

from which the K conversion coefficient of 412 keV transition is given by

$$\alpha_K = \frac{A_K(1+I_{1087}/I_{412})}{A_{966}/(1-x)-(A_K+A_{L+M+N})}. \tag{2}$$

Using the measured A_K, A_{966} and A_{L+M+N} and the values of Keeler and Connor (1965) for $x(0{\cdot}0124)$ and $I_{1087}/I_{412}(0{\cdot}0028)$, α_K is calculated to be $(301 \pm 3)\times 10^{-4}$. The error quoted is obtained from the separate errors in the areas of the peak and the beta continuum. The agreement of the present value with theoretical values (Rose 1958, Sliv and Band 1956) is very good. The results due to various experimental workers on the shape factor and K conversion coefficient of the 412 keV transition of ^{198}Au are summarized in table 2. Most of these results are consistent with a shape factor of the form $C(W) = k(1+aW)$. The measured degree of polarization (Van Klinken 1964) of this transition is compatible with $(-v/c)$. Hence the term b in the shape factor expression is zero. The W^2 term in the experimental shape factor of Hamilton *et al* (1962), Newbolt and Hamilton (1965) and Parsignault (1965) arises due to the improper choice of W_0. The two recent results (Paul 1965, Beekhuis and De Waard 1965) are in full agreement with the present determination and these exclude the earlier results (Wapstra *et al* 1958, de Vries *et al* 1960, Hamilton *et al* 1962, Lehmann 1964) and lead to values of $\alpha_{412K} = (299 \pm 4)\times 10^{-4}$ (Keeler and Connor), $(299 \pm 2)\times 10^{-4}$ (Paul), and $(301 \pm 3)\times 10^{-4}$ (present work), from the use of the PBS method in close agreement with theory (302×10^{-4}). This clearly demonstrates that the disagreement between the IEC and PBS methods comes mainly through the shape factor and the use of correct shape factors leads to agreement between the two.

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